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Effectiveness of Hotelling's T^2 Control Chart for Concrete Quality Monitoring

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ABSTRACT

In the world of construction, concrete quality plays a very important role because it directly affects the strength and durability of building structures. The objective of this study is to evaluate the efficiency of using the T^2 Hotelling control chart method in controlling concrete quality at PT. Wijaya Karya Beton Tbk by analyzing three main concrete quality parameters: slump, compressive strength, and tensile strength, based on data collected from January to February 2025. The methods employed include multivariate analysis approaches, such as correlation tests, multivariate normality tests, and Johnson transformations to correct non-normally distributed data, as well as the application of general variance control charts and T^2 Hotelling control charts. This study is unique compared to previous studies because it specifically applies the Hotelling T^2 method to the precast concrete industry, utilizing a more comprehensive analysis process. This approach not only demonstrates the effectiveness of this method but also strengthens the continuous quality control system. The research results show a significant correlation between concrete quality variables; however, the initial data did not meet the assumption of multivariate normality, necessitating the Johnson transformation, which proved effective in improving data distribution and enabling the application of Hotelling T^2 analysis. Based on the control chart, most observations remained within the established control limits; however, some samples were outside the limits, indicating process disturbances. Overall, this study concludes that the Hotelling T^2 method is effective in detecting process nonconformities at an early stage, thereby laying the foundation for continuous improvement in concrete production quality and making a significant contribution to strengthening the quality control system in the national construction sector.

Keywords: Aggregate Variance; Concrete Quality; Johnson Transformation; Statistical Process Control; T^2 Hotelling.

INTRODUCTION

In the modern construction industry, concrete quality plays a fundamental role in determining the success of a construction project. Concrete not only serves as the primary structural material but also determines the long-term durability and safety of a building. Non-compliance with concrete quality standards can lead to serious consequences, such as premature cracking, structural deformation, or even structural collapse, which could result in significant material losses and endanger human safety (Hamdi et al., 2022). Therefore, concrete quality control must be conducted properly to ensure that safety and structural reliability standards are maintained.

As a state-owned company and the biggest precast concrete producer in Southeast Asia, PT. Wijaya Karya Beton Tbk (WIKA Beton) bears a significant responsibility in maintaining the consistency of its product quality. Supervision is conducted on several key parameters, including slump (concrete mix consistency), compressive strength (maximum load-bearing capacity), and tensile strength (compressive strength). These three variables are interrelated and serve as primary indicators for evaluating concrete quality. Therefore, the quality control methods employed must be capable of addressing the interdependencies between variables simultaneously and comprehensively.

One of the statistical approaches widely used in monitoring the stability of multivariate processes is Statistical Process Control (SPC), particularly through the T^2 Hotelling control chart. This method is an extension of the Shewhart control chart, designed to detect process shifts based on a combination of several correlated quality variables (Mufidah, 2019). Its advantage lies in its ability to identify small changes that are often undetected when variables are analyzed separately (univariate).

However, the application of the T^2 Hotelling control chart has an important prerequisite: the data must be normally distributed and free from extreme outliers. Failure to meet this requirement can reduce the accuracy of the analysis results. To address this, Johnson's Transformation can be used to alter the data distribution to approach normality and meet the basic assumptions of multivariate analysis (Chou, Polansky, & Mason, 1998).

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During the evolution of Statistical Quality Control (SQC), conventional Shewhart control charts are generally used to monitor one quality characteristic at a time. However, this approach is limited in production systems involving complex interactions between variables. For such conditions, multivariate approaches such as Hotelling's T^2 control charts are more recommended. (Montgomery, 2013) emphasizes that the T^2 Hotelling chart is effective for controlling processes with more than one correlated variable, by quantifying the deviation of individual observations from the multivariate mean (mean vector). If the T^2 value exceeds the upper control limit (Upper Control Limit), the process is deemed uncontrolled and requires corrective action.

The effectiveness of this method has been proven in various studies. For example, (Arista et al., 2021) demonstrated that the T^2 Hotelling chart can detect deviations in the wheat flour production process (moisture, ash, and gluten). The study by (Wijayaningrum et al., 2022) at Roemani Muhammadiyah Hospital in Semarang also showed that all T^2 values from the five service dimensions were below the upper control limit, indicating no process deviations. Meanwhile, Sandria (2024) proved that the bootstrap-based T^2 Hotelling control chart is more effective in detecting deviations in paper pulp production when data do not meet the assumption of multivariate normality, with a smaller Average Run Length (ARL) in the bootstrap T^2 approach compared to the bootstrap sample.

Although it has been applied in various industrial sectors, specialized research highlighting the application of T^2 Hotelling in the precast concrete industry is still very limited. There has been no specific research applying T^2 Hotelling to the concrete industry in Indonesia. Therefore, this study was conducted to fill this gap, focusing on evaluating the effectiveness of the T^2 Hotelling control chart in monitoring concrete quality based on three main parameters: slump, crushing load, and crushing stress in the production process at PT. Wijaya Karya Beton Tbk. This study covers the period from January to February 2025, intending to contribute to the development of more effective and sustainable quality control methods in the precast concrete industry.

LITERATURE REVIEW

Statistical Quality Control (SQC) is a statistical method used to monitor and control the quality of manufactured products. Through the analysis of data collected during the production process, companies can ensure that their products meet established standards. SQC helps identify deviations or defects in products and determine their causes. The goal is to evaluate the production process and determine whether the products produced are acceptable or not. By implementing SQC, companies can prioritize the most effective improvements to reduce product defect rates. To measure quality, some commonly used tools include Check Sheets to record production data and defects, Histograms to visualize defect comparisons, and Control Charts to determine whether defects are still within normal control limits, i.e., between the Upper Control Level (UCL) and Lower Control Level (LCL), with the Control Level (CL) as the midpoint (Supardi & Dharmanto, 2020).

Quality control is a monitoring process carried out before, during, and after production to ensure that products meet established standards. This activity includes regulating raw materials to final products through inspection and comparison with standards, as well as analyzing causes of deviations, to improve quality and maintain product quality in accordance with company specifications (Alifka & Apriliani, 2024).

Control charts are a statistical method used as a measuring tool to monitor product quality. Montgomery explains that control charts serve as a medium for visualizing statistical analysis that can be performed using various approaches. The sample data collected is used to create control charts. If the sample values are within the control limits and do not show any particular systematic pattern, then the process is considered to be under control according to the level indicated by the chart (Trihandini, 2022).

Correlation Test

This study uses Pearson's correlation test, also known as product-moment, to analyze the data. This test is a simple form of correlation involving one dependent variable and one independent variable (Ardhaneswari & Suwitra, 2024). Correlation is used to measure the relationship between two variables and is a key requirement in the application of Hotelling's T^2 control chart. Correlation tests, particularly those using Pearson's coefficient, are used in statistical quality control to confirm the existence of a relationship between variables (Ramdani et al., 2023).

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$$r_{X_jX_k} = \frac{n\sum_{i=1}^{n} X_{ij}X_{ik} - \sum_{i=1}^{n} X_{ij}\sum_{i=1}^{n} X_{ik}}{\sqrt{n\sum_{i=1}^{n} X_{ij}^2 - \left(\sum_{i=1}^{n} X_{ij}\right)^2}\sqrt{n\sum_{i=1}^{n} X_{ik}^2 - \left(\sum_{i=1}^{n} X_{ik}\right)^2}} \;;\; -1 < \mathrm{r} < 1 \tag{1}$$

After obtaining the sample correlation matrix, the next step is to test the correlation between quality characteristics using Pearson's correlation. The hypothesis and test statistics used in this correlation test can be expressed in the following equation:

$$\chi^2_{hitung} = -\left[n - 1 - \frac{2p+5}{6}\right]\ln\left|\mathbf{R}\right| \tag{2}$$

Where:
p = number of quality characteristics
n = the total frequency of recorded observations
R = the correlation matrix depicting relationships among quality characteristic variables
with Hypothesis:
H_0: $\mathbf{R} = \mathbf{I}$ (there is no statistically significant correlation between the variables)
H_1: $\mathbf{R} \neq \mathbf{I}$ (there is a statistically significant correlation between the variables)
Testing Criteria:
H0 is rejected if $\chi^2_{hitung} > \chi^2_{\left(\alpha, \frac{m(m-1)}{2}\right)}$ This means that there is a correlation between variables.

Normal Distribution Test

The Multivariate Normal Distribution Test is applied to verify that the data satisfy the assumptions of a multivariate normal distribution, a requirement crucial for conducting multivariate analysis and process capability assessment. The hypotheses tested are:
H_0: The dataset is consistent with the assumptions of a multivariate normal distribution.
H_1: The dataset fails to meet the assumptions of a multivariate normal distribution.
To test this hypothesis, the following values are examined (Lestari & Rahmi Hg, 2023):

$$d_i^2 = \left(\mathbf{X}_i - \bar{\mathbf{X}}\right)^T S^{-1}\left(\mathbf{X}_i - \bar{\mathbf{X}}\right) \tag{3}$$

The steps in multivariate normality testing include:
1. Calculate the value according to the calculation in Equation (3).
2. Sort the values from smallest to largest.
3. Determine the value using the chi-square table.
4. Create a scatter plot that maps the pairs.

Data is said to be multivariate normally distributed if at least 50% of the values are based on the chi-square table.

Johnson Transformation

Data transformation is the process of converting data to another measurement scale. One method used in this study is the Johnson transformation, which offers greater flexibility than the Box-Cox transformation because it can handle data with a wide range of values, including bounded and unbounded data. The Johnson distribution system consists of three types of distributions: bounded (SB), lognormal (SL), and unbounded (SU), making it more adaptive to the characteristics of the original data distribution. The selection of the best distribution type in the Johnson transformation is performed optimally based on the highest z-value from the Shapiro-Wilk test.

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Table 1. Johnson's Transformation

The Johnson Family	**Transformation function**	**Range**
SB	$\gamma+\eta\ln\left[\left(x-\varepsilon\right)/\left(\lambda+\varepsilon-x\right)\right]$	$\eta,\ \lambda>0$ $-\infty<\gamma<\infty,$ $-\infty<\varepsilon<\infty,$ $\varepsilon<x<\varepsilon+\lambda$
SL	$\gamma+\eta\ln\left(x-\varepsilon\right)$	$n>0,$ $-\infty<\gamma<\infty,$ $-\infty<\varepsilon<\infty,$ $\varepsilon<x$
SU	$\gamma+\eta Sinh^{-1}\left[\left(x-\varepsilon\right)/\lambda\right]$ $where$ $Sinh^{-1}\left(x\right)=\ln\left[x+sqrt\left(1+x^{2}\right)\right]$	$\eta,\ \lambda>0,$ $-\infty<\gamma<\infty,$ $-\infty<\varepsilon<\infty,$ $-\infty<x<\infty$

Hotelling's T² Control Chart

The Hotelling T² control chart functions as a multivariate quality control instrument for monitoring the mean values of processes. This chart can be applied to both individual observation data and data grouped into subgroups (Salsabila & Wibawati, 2025). Hotelling's T² control chart for individual data depicts the distance of each data point from the center of the distribution. The following test statistics will be used (Sari & Widyasari, 2024):

$$T_i^2=\left(\mathbf{X_i}-\bar{\mathbf{X}}\right)'\mathbf{S}^{-1}\left(\mathbf{X_i}-\bar{\mathbf{X}}\right) \tag{4}$$

Where:

$\mathbf{X_i}$: Observation vector sample i

$\bar{\mathbf{X}}$: Average vector for each variable

$\mathbf{S}^{-1}$: Covariance matrix

With control limits:

$$\text{BKA}=\frac{p\left(m-1\right)\left(n-1\right)}{mn-m-p+1}F_{\alpha,p,mn-m-p+1}$$

$$\text{BKB}=0$$

Generalized Variance

Multivariate process control requires attention to two main aspects, namely control of the process mean and control of its variability. Variability in this process is represented by the covariance matrix, Σ. The elements situated on the principal diagonal of the matrix denote the variances of the respective variables, while the elements outside the diagonal indicate the covariance values between variables. There are two methods for controlling process variability: using univariate control charts $\mathbf{S}^2$ and generalized variance $\left|\mathbf{S}\right|$. The generalized variance control chart employs the determinant of the sample covariance matrix as a measure of dispersion within a multivariate framework (Hamidah et al., 2025).

$$E\left(\left|\mathbf{S}\right|\right)=b_1\left|\Sigma\right| \tag{5}$$

And

$$V\left(\left|\mathbf{S}\right|\right) = b_2\left|\Sigma\right|^2 \tag{6}$$

Where

$$b_1 = \frac{1}{(n-1)^p}\prod_{i=1}^{p}(n-i) \tag{7}$$

And

$$b_2 = \frac{1}{(n-1)^{2p}}\prod_{i=1}^{p}(n-i)\left[\prod_{j=1}^{p}(n-j+2)-\prod_{j=1}^{p}(n-j)\right] \tag{8}$$

So, the control limits for $\left|\mathbf{S}\right|$ are:

$$\text{BKA} = \left|\Sigma\right|\left(b_1 + 3b_2^{1/2}\right)$$

$$\text{Garis Tengah} = b_1\left|\Sigma\right|$$

$$\text{BKB} = \left|\Sigma\right|\left(b_1 - 3b_2^{1/2}\right)$$

Description:
Σ : Covariance matrix in the population.
$\mathbf{S}$: Covariance matrix in the sample.
p : Number of quality characteristic variables.
n : Number of samples in each subgroup.

METHOD

The study utilizes a quantitative approach, employing descriptive analytical methods to examine the data. This study is non-experimental, as it does not involve variable manipulation, but rather analyzes existing secondary data to evaluate the quality of concrete at PT. Wijaya Karya Beton Tbk.

This study uses secondary data from the concrete production process at PT Wijaya Karya Beton Tbk. The data analyzed includes 72 samples of concrete quality parameters collected during the period from January to February 2025. This study aims to evaluate the concrete quality control process at the company based on this data.

The three main variables analyzed in this study are:
Slump: Measures the consistency of concrete mix, which affects workability and concrete density.
Compressive Load: Measures the resistance of concrete to the maximum load it can withstand before sustaining damage.
Crush Stress: Measures the compressive strength of concrete, which is important in assessing the overall quality of concrete.

The analysis employed multivariate analysis techniques to examine the relationships between concrete quality variables. Pearson's correlation test was applied to measure these relationships, while a multivariate normality test was conducted to verify that the data satisfied the normal distribution assumption before implementing the multivariate control method. For data distributions that did not meet this assumption, a Johnson transformation was performed to correct them. Finally, Hotelling's T^2 Control Chart was applied to monitor the stability of the production process by simultaneously evaluating process mean and variability.

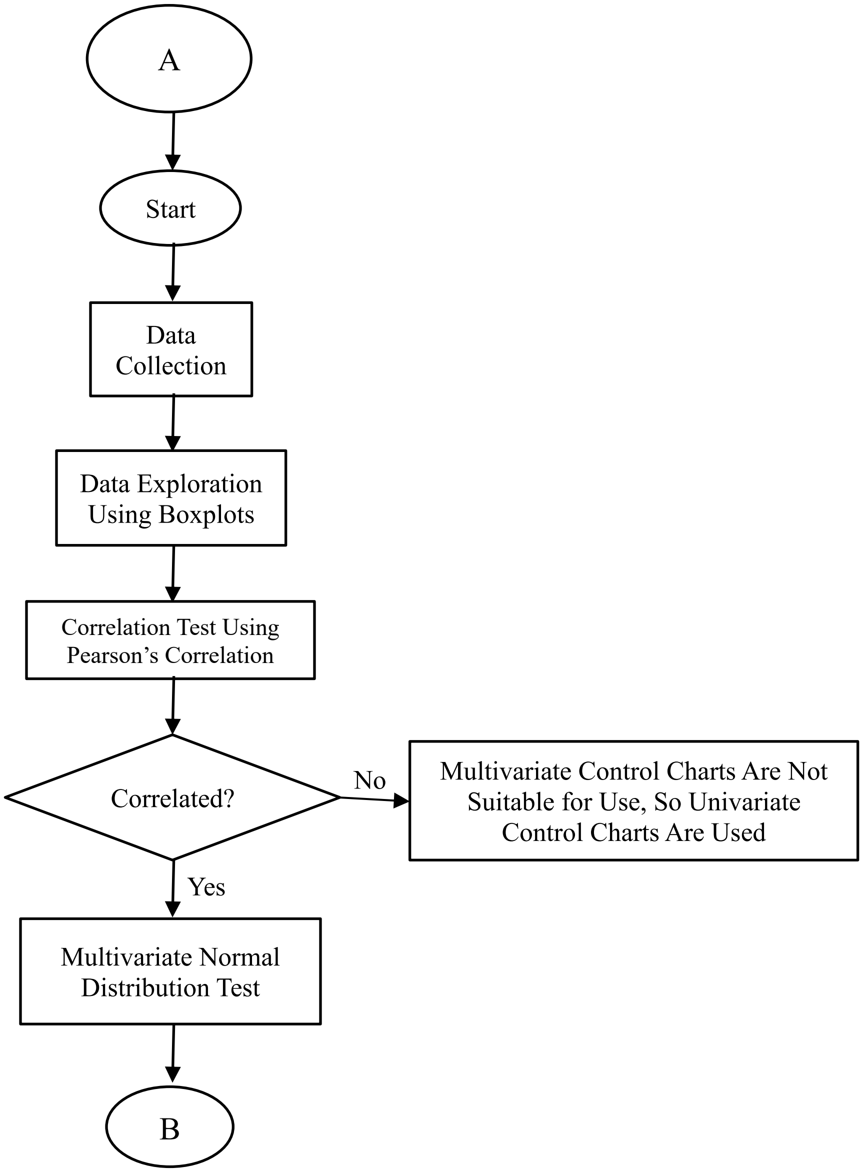

Figure 1. Flowchart

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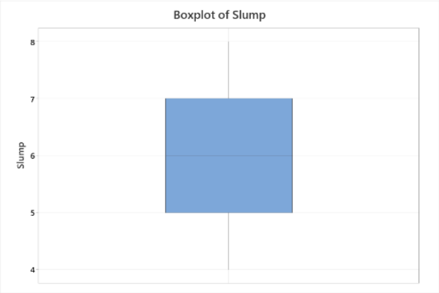
RESULT

Descriptive Statistics



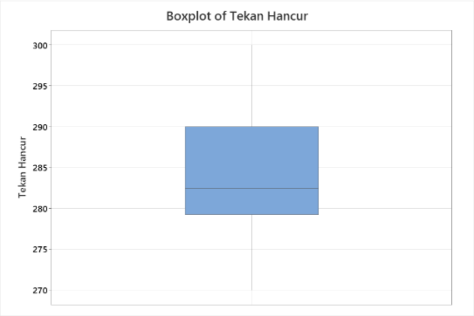


Figure 2. Boxplot on Slump Variable Figure 3. Boxplot On The Crushing Load Variable

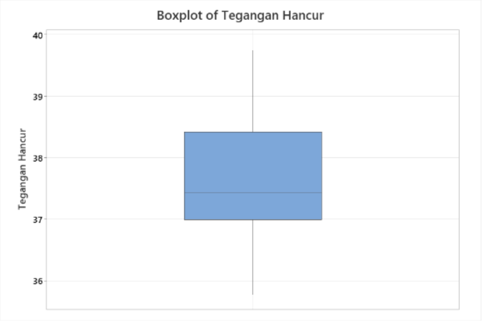


Figure 4. Boxplot on Crushing Stress Variable

Boxplots use a vertical axis to display data distribution, with the box representing the first quartile (Q1), median, and third quartile (Q3). The "whisker" lines indicate the spread of data outside the quartiles, while outliers are marked with special symbols outside 1.5 times the interquartile range. In the boxplot for the variables slump, crushing load, and crushing stress, the distribution of slump data appears symmetrical, while the other two variables show an asymmetrical distribution.

Correlation Test

Table 2.

Correlation Coefficient

Variable	**Slump (X_1)**	**Crushing Load (X_2)**	**Crushing Stress (X_3)**
Slump (X_1)	1	0,149	0,150
Crushing Load (X_2)	0,149	1	1
Crushing Stress (X_3)	0,150	1	1

Table 2 shows that the correlation value between slump and crushing load is 0.149, while the correlation between slump and crushing stress is 0.150, both of which are significant at the $\alpha = 0.05$ level. This indicates that slump has a correlation relationship with both variables, although the correlations between the other variables are not significant, meaning they are not correlated with each other. Therefore, the use of the T^2 Hotelling control chart remains relevant because this method can accommodate data with variables that are correlated both partially and simultaneously.

A correlation test is performed to evaluate the presence and strength of the relationship between two variables, as well as to assess how strong and in which direction the relationship is, whether positive, negative, or uncorrelated. This test is useful for researchers in verifying hypotheses, understanding the relationship between variables, and as a basis for decision-making or further analysis, such as regression, without concluding a causal relationship.

In the correlation test of variables X_1, X_2, X_3, the Pearson correlation method is used, giving the sample correlation coefficient using equation (2).

$$
\begin{aligned}
r_{x_1x_2} &= \frac{n\sum_{i=1}^{n} X_{i1}X_{i2} - \sum_{i=1}^{n} X_{i1}\sum_{i=1}^{n} X_{i2}}{\sqrt{n\sum_{i=1}^{n} X_{i1}^2 - \left(\sum_{i=1}^{n} X_{i1}\right)^2}\sqrt{n\sum_{i=1}^{n} X_{i2}^2 - \left(\sum_{i=1}^{n} X_{i2}\right)^2}} \\
&= \frac{72(123336) - (433)(20488)}{\sqrt{72(2753) - (433)^2}\sqrt{72(5834554) - (20488)^2}} \\
&= \frac{8880192 - 8871304}{\sqrt{198216 - 187489}\sqrt{420087888 - 419758144}} \\
&= \frac{8888}{\sqrt{10727}\sqrt{329744}} \\
&= \frac{8888}{59473} = 0{,}149
\end{aligned}
$$

Complete information regarding the correlation coefficients between variables can be seen in the following correlation matrix:

$$
\begin{aligned}
r_x &= \begin{bmatrix} 1 & r_{x_1x_2} & \cdots & r_{x_1x_q} \\ r_{x_2x_1} & 1 & \cdots & r_{x_2x_q} \\ \vdots & \vdots & \ddots & \vdots \\ r_{x_qx_1} & r_{x_1x_q} & \cdots & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0{,}149 & 0{,}150 \\ 0{,}149 & 1 & 0{,}999 \\ 0{,}150 & 0{,}999 & 1 \end{bmatrix}
\end{aligned}
$$

To determine whether the correlation is statistically significant, the following hypotheses are used:
H_0: (no correlation between variables)
H_1: (There is a correlation between variables)

To determine the correlation value of concrete, a T-test is performed for each sample.

$$\bar{r}_{X_1} = \frac{1}{3-1}\left[0{,}1494 + 0{,}1497\right] = 0{,}14955$$

$$\bar{r}_{X_2} = 0{,}57465$$

$$\bar{r}_{X_3} = 0{,}5748$$

$$\bar{r}_X = \frac{2}{(3)(2)}\left[0{,}149 + 0{,}150 + 0{,}999\right] = 0{,}4327$$

$$
\begin{aligned}
\sum\sum_{k>j}\left(r_{X_k}r_{X_j} - \bar{r}_X\right)^2 &= \left(0{,}1494 - 0{,}4327\right)^2 + \left(0{,}1497 - 0{,}4327\right)^2 + \left(0{,}9999 - 0{,}4327\right)^2 \\
&= 0{,}0802589 + 0{,}080089 + 0{,}321716 \\
&= 0{,}4820639
\end{aligned}
$$

$$\begin{aligned}\sum_{j=1}^{p}\left(\bar{r}_{X_j}-\bar{r}_X\right) &= \left(0,14955-0,4327\right)^2+\left(0,57465-0,4327\right)^2+\left(0,5748-0,4327\right)^2\\ &= \left(0,0801739\right)+\left(0,0201498\right)+\left(0,0201924\right)\\ &= 0,1205161\end{aligned}$$

$$\begin{aligned}\hat{Y}_X &= \frac{\left(3-1\right)^2\left[1-\left(1-0,4327\right)^2\right]}{3-\left(3-2\right)\left(1-0,4327\right)^2}\\ &= \frac{\left(2\right)^2\left[1-0,3218\right]}{3-\left(1\right)\left(0,3218\right)^2}\\ &= \frac{4\left(0,6782\right)}{2,89644}\\ &= 0,9366\end{aligned}$$

Therefore, the T-test statistical value obtained is:

$$\begin{aligned}T &= \frac{\left(72-1\right)}{\left(1-0,4327\right)^2}\left[0,4821-\left(0,9366\right)\left(0,1205\right)\right]\\ &= 81,4594\end{aligned}$$

With, $\alpha = 0,05$; $v = \frac{\left(q+1\right)\left(q-2\right)}{2} = 2$ so the critical value for the concrete value is $\chi^2_{2;0,05} = 5,991$ It can be stated that $T > \chi^2_{2;0,05}$, so that it can be proven, indicating that the null hypothesis (H₀) is rejected, in other words, there is a significant correlation between the variables.

Normal Distribution Test

Multivariate normality testing is performed to ensure that the data meet the assumptions of multivariate normal distribution, which is the basis of multivariate analysis. The test is executed using the hypotheses specified as follows:
H_0: The dataset satisfies the assumptions of a multivariate normal distribution.
H_1: The dataset fails to meet the assumptions of a multivariate normal distribution.

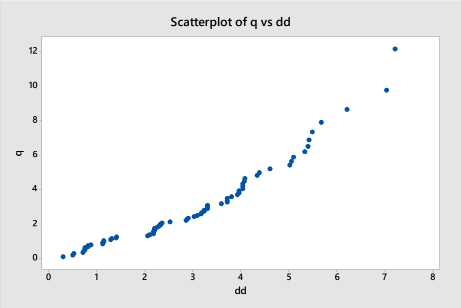


Figure 5. Normal Distribution Test

With a significance level of α = 0.05, the rejection criterion for H₀ is set when the proportion of data with squared distance values greater than the critical limit (2.366) exceeds 50%. The results show that 44% of the data are within the limits, while 56% (40 out of 72 points) are outside the ellipse limits, so H₀ is rejected. In conclusion, the slump, crushing load, and crushing stress data for January–February are not normally distributed multivariate.

Johnson's Transformation

The test results indicate that the assumption of normality is not upheld for the slump, crushing load, and crushing stress data collected from PT Wijaya Karya during the period January–February 2025. In response, the Johnson transformation was performed using Minitab 18 software.

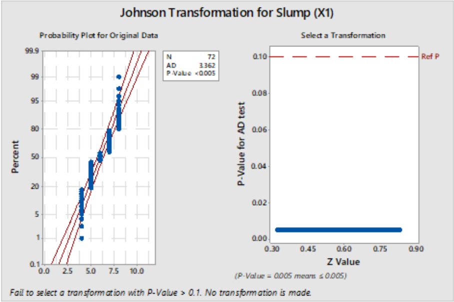


Figure 6. Johnson's transformation on slump variable (X_1)

Although Johnson's transformation model on the slump variable has been tested (Figure 5), no model produced a p-value > 0.1, so the transformation was considered unsuccessful in overcoming the abnormality of the data distribution in that variable. This failure indicates that the assumption of multivariate normality is not fully met, which can reduce the reliability of Hotelling's T^2 analysis due to an increased risk of false alarms and missed detections. Therefore, the interpretation of control chart results must be done carefully, including notes on limitations.

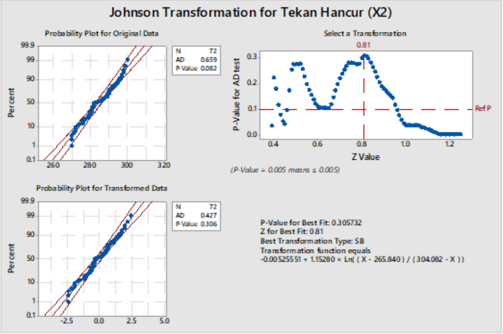


Figure 7. Johnson transformation of crushing load variable (X2)

Johnson's transformation analysis of the load failure variable is presented through three main graphs, namely the probability graph of the data before transformation, the probability graph after transformation, and the z-value and p-value graphs. As shown in Figure 6, the transformation was performed using the Johnson System of Distributions method. The analysis results indicate that the most appropriate transformation function for the load-bearing capacity variable successfully improves the fit to the normal distribution, as supported by the p-value fulfilling the established significance criteria.

$$x' = -0.0052555 + 11.15280 \times Ln\left(\frac{(X - 265.840)}{(304.082 - X)}\right)$$

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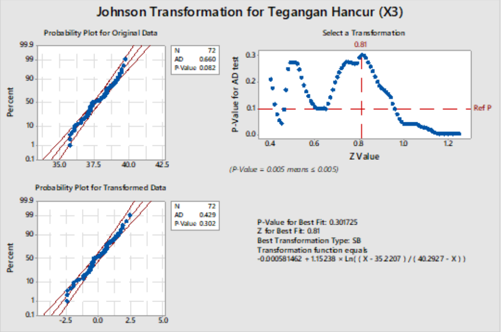

Figure 8. Jonson transformation on the crushed stress variable (X_3)

The Johnson transformation of the fracture stress variable was performed using the SB distribution method, one of the types in the Johnson System. This transformation model is shown in Figure 7, which consists of three main graphs: the probability graph before transformation, the probability graph after transformation, and the z-value and p-value graphs. The analysis results indicate that the SB transformation function is the most suitable model for the breakdown voltage variable, with a significant improvement in the normal distribution as indicated by the p-value meeting the significance criteria.

$$x' = -0.000581462 + 1.15238 \times Ln\left(\frac{(X - 35.2207)}{(40.2927 - X)}\right)$$

After performing Johnson's transformation on data that did not meet the normality assumption, multivariate normality testing of the transformed data was performed using the Minitab macro.

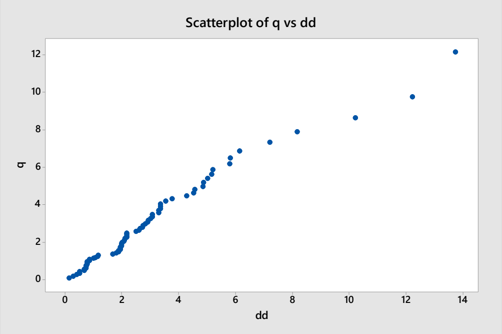

Figure 9. Normal Distribution Test After Johnson Transformation

The results show that 52.8% of the data has a distance less than the limit, so H_0 is accepted. Thus, it can be concluded that the slump, crushing load, and crushing stress variables for the January–February period at PT Wijaya Karya are normally distributed. After confirming normality, the data were then analyzed using the classic Hotelling's T^2 control chart.

Generalized Variance

Process variance control analysis is performed using a generalized variance control chart. The subsequent results present the analysis of variability in the concrete production process.

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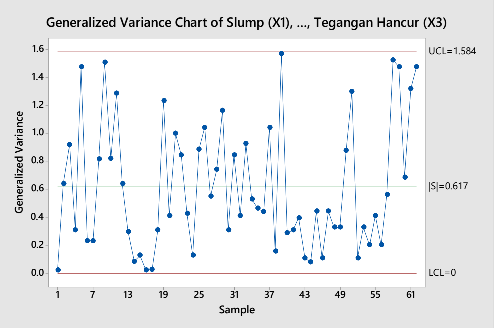

Figure 10. Generalized Variance Control Map Before Improvement

In the initial stage, a generalized variance control chart was used to monitor the variability of three concrete quality variables, namely slump, crushing load, and crushing stress, based on 72 samples. The mean value was recorded at 0.763, with an upper control limit (UCL) of 1.960 and a lower control limit (LCL) of 0. There were three points (samples 8, 63, and 68) that exceeded the upper limit and were marked with red boxes, indicating process uncontrollability or significant variance fluctuations. Although most points were within the control limits, these deviations required further analysis to identify the exact causes.

Subsequently, a step-by-step analysis was conducted through six improvement cycles.

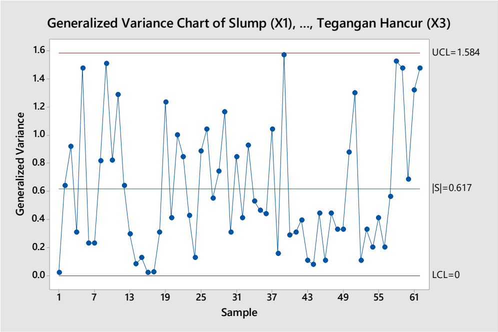

Figure 11. Generalized Variance Control Map After Improvement

After filtering and excluding points beyond the control limits, the remaining sample size was 62. During this process, the mean variance decreased progressively from 0.763 to 0.617, and the UCL decreased from 1.960 to 1.584, while the LCL remained at 0. Additional deviations were also found at several other points (samples 32, 52, 59, 60, and 61), but in the final iteration, all data points were within control limits, indicating that the variability process was statistically controlled.

After the variability process was declared controlled, the analysis continued with a Hotelling's T^2 control chart to monitor the stability of the process mean. This chart was constructed based on 62 samples with a covariance matrix from the previously transformed data. The upper control limit (UCL) was set at 18.93, with a median value of 3.68. The plot results show that all observation points, including the first point with $T^2 = 1.00$, are within the control limits. Therefore, it can be concluded that the average of the concrete production process is in a statistically controlled state.

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T^2 Hotelling

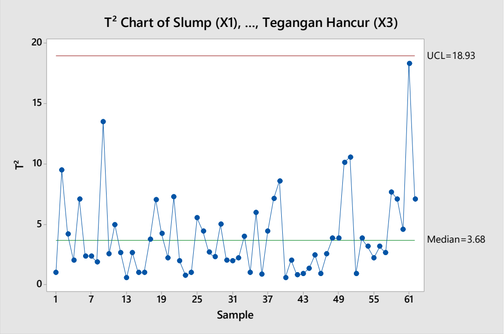

Figure 12. T^2 Hotelling Control Map without Improvement

After the variability process is declared controlled through generalized variance analysis, the next step is to evaluate the quality characteristics of concrete based on the process average using the T^2 Hotelling control chart. This chart monitors the stability of the production process from three main variables (slump, crushing load, and crushing stress), with a covariance matrix taken from data that has met the assumptions of statistical control. The following figure depicts the T^2 Hotelling control chart.

The T^2 Hotelling control chart consists of 62 data points, with the horizontal axis showing the number of observations and the vertical axis displaying the calculated T^2 values. The upper control limit (UCL) is 18.93, and the median observation value is 3.68. The first observation point has a T^2 value of 1.00. All data points lie within the control limits, indicating that the concrete production process is statistically controlled for the mean.

Causes of Non-Compliance

Non-compliance of concrete composition with company standards has an impact on the quality of concrete, which in turn affects its effectiveness, including its application in planting media. This decline in quality can reduce consumer confidence in the company. Based on the results of the analysis at PT Wijaya Karya, the main causes stem from human factors, such as negligence, lack of precision, and decreased concentration during the production process. In addition, machine damage due to a lack of maintenance also contributes to errors in the mixing of concrete composition.

DISCUSSIONS

The study results reveal that the application of the T^2 Hotelling control chart at PT Wijaya Karya Beton Tbk is effective in early detection of process disturbances, thanks to the significant relationship between slump, crushing load, and crushing stress. This multivariate approach is more appropriate than univariate because quality variables are interdependent. Initial normality tests showed that only 44% of the data met the multivariate normal distribution, so a Johnson transformation was performed. As a result, the distributions of crushing load and crushing stress improved, while the slump was not yet optimal; however, the overall proportion of normal data increased to 52.8%.

In the analysis conducted, although the slump variable could not be transformed into a normal distribution using the Johnson transformation ($p\text{-value} < 0.1$), the data as a whole is still considered valid. This is because data validity does not only depend on the normality of each variable (univariate), but on the combined distribution of all variables approaching multivariate normality.

After successfully transforming the crushing load and crushing stress variables, the proportion of data that met the multivariate normality requirement increased from 44% to 52.8%, which exceeds the minimum threshold of 50%. This shows that, although the slump variable itself is not normal, the relationship and combined distribution of these three variables are strong enough to meet the multivariate assumption. The conclusion from this finding is that multivariate analysis prioritizes the overall data structure. Therefore, a deficiency in one variable can be compensated for by improvements in other variables, although this limitation still needs to be considered as a potential bias in interpretation.

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Stability analysis of variability using generalized variance control charts identified three points outside the control limits. These were rectified through six stages of data filtering until all observations complied with the control limits. Furthermore, the classic T^2 Hotelling control chart showed that all 62 samples were below the upper control limit (18.93), indicating that the process average was under control.

This method proved superior in detecting deviations missed by conventional methods, enabling swift corrective actions before product quality deteriorates. However, its effectiveness depends on the fulfillment of the assumption of multivariate normal distribution. Besides technical factors like machine damage, human factors like negligence and lack of concentration also affect concrete quality.

The data distribution shows a relatively symmetrical slump, while the crushing load and crushing stress are asymmetrical and have outliers, indicating process fluctuations. The strong correlation between slump and crushing load, as well as crushing stress, confirms the interdependence of concrete quality. These findings align with previous research demonstrating the effectiveness of T^2 Hotelling in quality control across various industries.

The implication is that while most data is under control, the presence of samples outside the control limits indicates potential disruptions that need to be anticipated. Data transformation is also important to ensure the validity of the analysis. Overall, the routine application of T^2 Hotelling can enhance the consistency and quality of precast concrete sustainably.

Several limitations inherent in this study should be duly noted. The application of Hotelling's T^2 method requires multivariate normally distributed data, so that non-compliance with this assumption can reduce the accuracy of the results. This method is also sensitive to the presence of extreme values that can potentially affect the calculation of the mean and covariance. Additionally, the relatively small sample size of 72 data points and the 2-month data collection period (January to February 2025) may diminish the precision of parameter estimation and the determination of control limits. The use of secondary data, in the form of company documentation (concrete laboratory test results) rather than direct observation or experimentation by the researcher, further limits the researcher's control over the quality, consistency, and completeness of the information analyzed. The findings of this study are only applicable to a specific type of concrete product and period, so their application to different periods or product types should be done with caution.

In my research, the T^2 Hotelling control chart was applied as the primary method to monitor the stability of the concrete production process in a multivariate manner, following tests of correlation, normality, and Johnson transformation. The results indicated that most observation points were within control, although two outliers exceeded the upper control limit, requiring further investigation into the causes of process disturbances. In comparison, (Hamidah et al., 2025) examined the quality control of industrial sugar wastewater using the T^2 Hotelling control chart after ensuring that the multivariate variance process was stable through the generalized variance chart. Their findings showed that the process was already under control in the first test, with a UCL value of 17.49, confirming that the wastewater process was maintained within statistical limits. The key distinction between the two studies lies in the sequence of method application: my research employs the T^2 Hotelling chart from the beginning of the analysis, whereas (Hamidah et al., 2025) applied it after variance stabilization, meaning the process was already in a stable condition before multivariate mean analysis. The uniqueness of my study lies in its application to the precast concrete industry in Indonesia, specifically at PT Wijaya Karya Beton Tbk, the largest producer in Southeast Asia. By focusing on the quality control of precast concrete, this study utilizes the T^2 Hotelling multivariate control chart to simultaneously analyze key variables, namely slump, compressive load, and compressive strength. Furthermore, the use of Johnson transformation addresses non-normal data, allowing for more accurate analysis. Consequently, this research not only provides a more comprehensive approach but also contributes practical insights for enhancing the quality of precast concrete in support of national infrastructure development.

CONCLUSION

Based on research into the concrete production process at PT Wijaya Karya Beton Tbk using the Statistical Process Control (SPC) method with the T^2 Hotelling control chart, it was concluded that this method is effective in simultaneously monitoring the stability of the process for three concrete quality variables, namely slump, crushing load, and crushing stress. Before the analysis was conducted, the Johnson Transformation was used to ensure the data met the assumptions of multivariate normal distribution. Process variability was monitored through generalized variance maps, which initially showed uncontrolled conditions with a mean variance of 0.763, UCL = 1.960, and LCL = 0, where 3 out of 72 samples (4.2%) exceeded the control limits. After six cycles of improvement, the variance decreased to 0.617 with a reduced UCL = 1.584, and 100% of the 62 remaining samples were within control limits.

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Furthermore, the Hotelling's T^2 control chart, with UCL = 18.93 and median $T^2 = 3.68$, showed that all observation points (100%) were within control limits, confirming that the process mean was statistically stable. These results indicate that the T^2 Hotelling test enables early detection of deviations and provides a reliable tool for maintaining concrete quality. Based on these findings, the method is recommended as a quality control tool in the precast concrete industry and has the potential for further development through multivariate process capability analysis. Future research is suggested to explore the integration of T^2 Hotelling with the multivariate capability index or the application of machine learning techniques for predictive quality control in concrete production.

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