

Metric Space Based Product Portfolio Optimization For Msmes Using Genetic Algorithm

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ABSTRACT

Micro, Small, and Medium Enterprises (MSMEs) face capital allocation challenges across multi-category product portfolios in the digital era, while the classical Mean-Variance Optimization (MVO) model exhibits structural limitations due to normality assumptions, estimation instability on small samples, and its inability to capture more than two risk dimensions. This study developed and validated an MSME product portfolio optimization model based on metric space theory and genetic algorithm, capable of handling multidimensional operational risk and cardinality constraints. A weighted distance function was constructed on a three-dimensional feature space comprising contribution margin (μ), sales coefficient of variation (σ), and Margin of Safety (MOS). Metric space validity was proved deductively through verification of non-negativity, symmetry, and triangle inequality axioms. The genetic algorithm was designed with a three-step repair operator to satisfy allocation and cardinality constraints. The model was validated using secondary data from 10 product categories over a six-month period (April–September 2025) and benchmarked against Equal Weighting (EW) and MVO strategies. Formal proof and empirical verification on 1,000 data triplets yielded zero axiom violations. The genetic algorithm achieved the highest fitness value (0.71798), outperforming EW by 10.3% and MVO by 18.6%. The optimal portfolio selected four products: paper (28.2%), books (21.8%), measuring tools (29.0%), and electronics (20.9%), geometrically explained by inter-product distances in the distance matrix (d_w) ($P_3, P_{10} = 0.8649$). The proposed model is mathematically rigorous and computationally superior in the validation case. Although constituting a proof-of-concept on a single secondary dataset, this framework offers potential for development into an MSME capital allocation decision support system.

Keywords: Metric space; Genetic algorithm; MSME portfolio; Margin safety; Cardinality constraint

INTRODUCTION

Micro, small, and medium enterprises (MSMEs) constitute one of the main pillars of the Indonesian economy, contributing more than 60% of Gross Domestic Product and absorbing the majority of the national workforce [1]. The growth of the digital economy, with internet penetration reaching 78.9% of the population and more than 30 million MSME operators having entered the digital ecosystem [2], has pushed MSMEs to manage an increasing number of product categories through various online marketplace platforms. This shift has created a new challenge: determining the optimal allocation of working capital across product categories that differ in their return and risk characteristics.

In practice, MSME capital allocation decisions are still largely based on owner experience or simple historical observation [3]. This approach treats each product category in isolation and struggles to produce optimal outcomes, because it cannot accommodate the multidimensional characteristics inherent to each product contribution margin, demand stability, and sales safety among them [4]. High volume products do not necessarily yield the best return, while high-margin products often carry more volatile demand, so a poorly informed allocation can result in inefficient use of resources and foregone profit [5]. The underlying

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problem this study addresses is the absence of a quantitative framework capable of integrating these operational dimensions simultaneously into a single capital allocation decision, rather than evaluating each product category separately.

Addressing this problem calls for a shift from intuition-based allocation toward an optimization framework grounded in established theory. Modern portfolio theory, introduced by Markowitz (1952), offers a natural starting point, as it formalizes the return-risk trade-off mathematically through the mean-variance model. However, applying this model directly to the MSME context runs into three structural limitations. First, the model relies on the assumption of normally distributed returns and stable covariance estimation, conditions that MSME sales data frequently fail to meet due to fat tails and skewness [6]. Second, risk is represented solely through variance, which overlooks contribution margin and Margin of Safety both critical for physical-product MSMEs. Third, optimization outcomes are highly sensitive to estimation error, where small parameter shifts can substantially alter the optimal portfolio composition [7], [8], a problem compounded by the short sales histories typical of MSMEs, which further destabilize covariance estimation [9], [10].

Beyond these statistical concerns, risk in MSME product portfolios does not always manifest as return volatility. Two product categories may individually exhibit low sales variance yet still create risk concentration if both depend on the same market segment or demand pattern. This has motivated the development of distance-based portfolio optimization, which treats diversification benefit as a function of dissimilarity rather than co-movement of returns between assets [11], [12]. Metric space theory provides an appropriate mathematical foundation for this approach: by representing each product category as a point in a multidimensional feature space, the degree of dissimilarity between products can be measured through a distance function satisfying the axioms of non-negativity, identity of indiscernibles, symmetry, and the triangle inequality, without requiring any assumption about the underlying probability distribution or covariance structure of the data.

A further complication arises from a practical constraint: MSME owners can realistically manage only a limited number of product categories at once, so any workable model must cap the number of categories selected. This cardinality constraint renders the optimization problem nonlinear and combinatorial, making classical gradient-based methods ineffective. A genetic algorithm (GA) is used to address this structure, given its demonstrated capacity for global exploration of the solution space, its ability to accommodate discrete constraints directly, and its suitability for multi-criteria objective functions advantages that have been shown effective in portfolio optimization problems with comparable constraints [13].

Although research on distance-based portfolio optimization and genetic algorithms has grown substantially, most of this work remains concentrated on financial assets. To date, research that develops a metric-space-based risk measure for MSME product portfolios—one that draws on operational indicators as a proxy for product risk and integrates this measure with a genetic algorithm to handle cardinality constraints—remains very limited.

Based on this background, this study aims to develop and validate an MSME product portfolio optimization model grounded in metric space theory and genetic algorithm. The main contributions of this research are: (1) the development of a weighted distance function over a three-dimensional feature space contribution margin, sales stability, and Margin of Safety that satisfies the metric space axioms; (2) the formulation of a metric-space-based portfolio risk measure that requires no distributional assumptions; (3) the development of a multi-criteria optimization model that integrates profitability and diversification under a cardinality constraint, solved using a genetic algorithm; and (4) the validation of the model using empirical data from an MSME in the stationery sector, covering the April–September 2025 period [14] benchmarked against Equal Weighting and conventional Mean-Variance Optimization strategies.

The proposed model is expected to yield a distance function whose metric validity is confirmed both deductively and empirically, together with a genetic algorithm capable of producing portfolio allocations

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that outperform conventional benchmarks while remaining interpretable through the geometric structure of the resulting distance matrix. More broadly, this study is expected to contribute academically by bridging portfolio theory and operations research for non-financial assets, to provide MSME owners with a more structured and data-driven basis for capital allocation decisions, and to lay groundwork for further development into a decision support system for MSME capital allocation.

The remainder of this study organized into four sections. Section 2 describes the research method, covering the construction of the weighted distance function over the μ, σ MOS feature space, the deductive proof of the metric space axioms, the design of the genetic algorithm together with its three-step repair operator, and the validation procedure against the Equal Weighting and Mean-Variance Optimization benchmarks. Section 3 presents the results and discussion, including the empirical verification of the distance function on 1,000 data triplets, the fitness comparison across the three allocation strategies, and the geometric interpretation of the resulting optimal portfolio through the distance matrix. Section 4 concludes of this study, summarizing the main findings, acknowledging the limitations of a proof-of-concept study based on a single secondary dataset, and outlining directions for developing the framework into an MSME capital allocation decision support system.

LITERATURE REVIEW

The mean-variance optimization (MVO) framework introduced by Markowitz (1952) remains the dominant approach to portfolio allocation under risk. However, recent studies have highlighted several structural limitations that reduce its effectiveness outside ideal financial settings. MVO relies on the assumption of normally distributed returns, represents risk solely through variance, and is highly sensitive to estimation errors in expected returns and covariance matrices, particularly when only limited historical data are available[6], [7], [8], [9], [10] . These limitations are especially relevant for micro, small, and medium enterprises (MSMEs), where only short operational histories are typically available. Nevertheless, existing studies remain confined to financial assets and do not investigate whether similar issues arise when the portfolio consists of product categories within a single enterprise.

To overcome these limitations, recent research has explored distance-based diversification as an alternative to covariance-based risk modeling. Studies by [15] and [11] demonstrate that metric based distance structures can capture diversification opportunities beyond linear correlation, while classical metric space theory [16] provides a rigorous mathematical foundation that is independent of distributional assumptions. Despite these advances, existing distance-based approaches still construct feature spaces from asset price or return series. To the best of the authors' knowledge, no previous study has developed a metric-space-valid distance function using operational indicators available to MSMEs, such as contribution margin, demand stability, and margin of safety.

From an optimization perspective, genetic algorithms (GAs) have proven effective for solving cardinality-constrained portfolio problems that are difficult for conventional optimization methods [13], [17], [18]. Meanwhile, MSME studies have established the practical importance of operational indicators such as break-even point and margin of safety, yet these indicators have not been integrated into a portfolio allocation framework. This study addresses these gaps by proposing a metric-space-based product portfolio model that employs a weighted distance function derived from MSME operational indicators, a distribution-free risk measure, and a cardinality constrained GA optimization framework. The proposed approach is validated empirically against Equal Weight (EW) and Mean Variance Optimization (MVO) benchmarks.

METHOD

This study develops MSME product portfolio optimization model built on two mathematical foundations: metric space theory, which provides the risk-measurement layer, and a genetic algorithm, which provides the solution mechanism for the resulting constrained optimization problem. The model is

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organized into three hierarchically connected layers: product representation, risk measurement, and portfolio optimization. The overall research flow is summarized in Figure 1. The choice of a distance-based, metric-space risk measure over the conventional covariance-based approach is grounded in recent evidence that distance structures between assets can yield better diversification outcomes than classical covariance methods[15] [11], [12]). Formally, metric space theory offers an axiomatic definition of distance non-negativity, identity of indiscernibles, symmetry, and the triangle inequality that guarantees a mathematically consistent and unambiguous dissimilarity measure [16], and has been applied broadly in pattern recognition, data mining, and optimization for the same reason [19].

Economic interpretation. The value $d_w(\bar{v}_i, \bar{v}_j)$ measures the dissimilarity in risk profile between products i and j within the feature space. A large d_w value indicates that the two products have substantially different risk profiles, so allocating capital to both simultaneously provides a meaningful diversification benefit; a small value indicates near-identical profiles, offering little diversification benefit from combining the two.

Metric Based Portfolio Risk Measure

For an allocation vector $x = (x_1, \dots, x_n)$ with $\sum_i x_i = 1$ and $x_i \geq 0$, a diversification-based portfolio risk measure is defined as:

$$R_p(x) = \sum_{i=1}^n \sum_{j=1}^n x_i \cdot x_j \cdot d_w(\bar{v}_i, \bar{v}_j) = x^T D x \quad (4)$$

where D is an $n \times n$ distance matrix with elements $D_{ij} = d_w(\bar{v}_i, \bar{v}_j)$. $R_p(x)$ represents the weighted average distance across all product pairs in the portfolio; a higher R_p indicates that the selected products differ substantially in risk profile, and the portfolio is therefore better diversified. This measure is fundamentally distinct from a conventional covariance-based risk model, as it requires no assumption about the statistical distribution of the underlying product data a property of practical importance given the limited historical data typically available to MSMEs. Because the matrix D is not generally positive semidefinite, $R_p(x)$ is non-convex in x , which motivates the use of a metaheuristic solver rather than a gradient-based method.

Multi Criteria Optimization Model

The proposed model maximizes an objective function that integrates portfolio return and metric-based risk simultaneously:

$$\max_x f(x) = \lambda \cdot \frac{Ret_p(x)}{\mu_{max}} + (1 - \lambda) \cdot \frac{R_p(x)}{D_{max}} \quad (5)$$

subject to:

$$(C1) \sum_{i=1}^n x_i = 1 \quad (6)$$

$$(C2) 0 \leq x_i \leq 0.40 \quad \forall i \quad (7)$$

$$(C3) |\{i: x_i > 0\}| \leq K \quad (8)$$

where $Ret_p(x) = \sum_i x_i \mu_i$ is portfolio return, μ_{max} and D_{max} are normalization factors that ensure both components are on a comparable scale, $\lambda \in [0,1]$ is an investor preference parameter, and K is the cardinality bound. Constraint (C1) ensures full capital allocation, (C2) prevents excessive concentration in a single product, and (C3) limits the number of active products in the portfolio. Constraint (C3) renders the problem combinatorially NP-hard, so that classical gradient-based methods cannot guarantee a global solution — which justifies the use of a genetic algorithm as the solution method.

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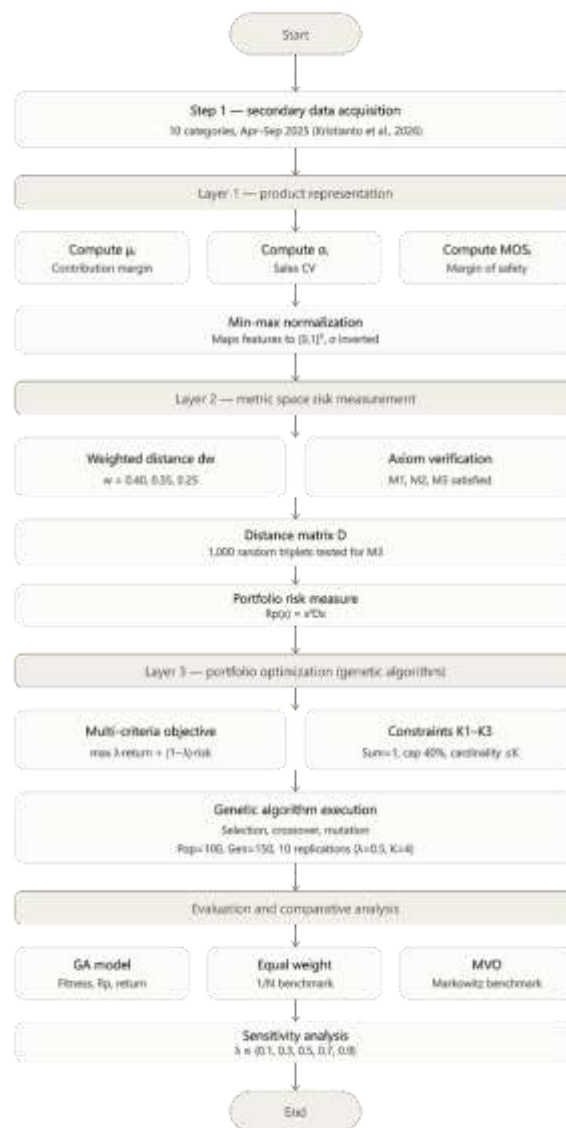


Figure 1. Research flow diagram

Genetic Algorithm

A genetic algorithm (GA) is an evolutionary optimization method, that uses selection, crossover, and mutation to iteratively generate improved solutions. In portfolio optimization research, GA has been shown to handle nonlinear objective functions and discrete constraints that are difficult for classical optimization methods to resolve. During selection, individuals with higher fitness values have a greater probability of being chosen as parents; crossover combines information from two parent solutions to produce new candidate solutions; and mutation preserves population diversity and helps prevent premature convergence to a local optimum. [20]. GA remains one of the most widely used metaheuristics for problems of this kind, and its integration with machine learning and hybrid approaches continues to develop for improving solution quality and convergence speed [17], [18].

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In this study, GA is used to solve the multi-criteria optimization problem in Equations (5)–(8). Through iterative evolution of a population of candidate allocation vectors, GA explores the solution space broadly and directs the search toward a globally optimal allocation weight vector x^* representing the MSME product portfolio composition with minimum metric risk.

Data Source and Case Study

The model is validated using operational data from Stationery shop, an MSME operating in the stationery sector in Bekasi, covering the period April–September 2025 [14]. The dataset comprises 10 product categories, with a monthly fixed cost of Rp 11,570,000. For each product category, monthly sales data over the six-month period were used to compute the contribution margin, the coefficient of variation of monthly sales volume (as the sales-stability indicator), and the Margin of Safety, following Equations (1) and (2). These three indicators were then normalized as described to construct the feature vector $\tilde{v}_i = (\tilde{\mu}_i, \tilde{\sigma}_i, \tilde{MOS}_i)$ for each product category i .

Validation and Evaluation Procedure

Model evaluation was conducted in four stages, applied chronologically following model construction. First, the mathematical validity of the weighted distance function was verified empirically on the case data, in addition to the deductive proof. The distance matrix D was tested against axioms (M1)–(M3): all diagonal elements were checked for equality to zero with positive off-diagonal elements (M1); symmetry was checked via $D = D^T$ (M2); and the triangle inequality was tested on 1,000 randomly sampled triplets of products to check for violations (M3). Second, the genetic algorithm was run for 10 independent replications with parameters $\lambda = 0.5$ and $K = 4$, using a population size of 100 individuals, 150 generations, crossover probability $p_c = 0.85$, and mutation probability $p_m = 0.08$. Convergence was evaluated through the mean, maximum, minimum, standard deviation, and coefficient of variation of the resulting fitness values across replications, to confirm that the algorithm consistently converges to the same solution. Third, the resulting optimal portfolio was benchmarked against two comparison strategies: Equal Weighting (EW), which allocates capital uniformly across all product categories, and conventional Mean-Variance Optimization (MVO). The three approaches were compared on fitness value $f(x)$, portfolio return $Ret_p(x)$, and metric risk $R_p(x)$, allowing the trade-off between return and diversification under each strategy to be assessed directly. Fourth, a sensitivity analysis was conducted by varying the preference parameter $\lambda \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, to observe how the optimal portfolio composition and its performance shift as investor preference moves from a diversification-oriented to a return-oriented objective.

RESULT

Feature Space Construction and Distance Matrix

The three dimensional feature vectors for the ten product categories of Stationery shop, computed following Equations (1)–(2) and normalized. The table 1 below are presented the normalized feature vectors of product categories.

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Table 1. Normalized Feature Vectors of 10 Product Categories

Code	Category	μ_i (%)	σ_i (CV)	MOS_i (%)	μ_i	$\bar{\mu}_i$ (inv)	$\overline{MOS}_i$
P01	Stationery	21.74	0.361	2.39	0.8240	0.4543	0.1345
P02	Eraser	20.00	0.323	2.81	0.5952	0.5322	0.1821
P03	Paper	15.48	0.158	10.14	0.0000	0.8747	1.0000
P04	Filing	20.97	0.097	5.60	0.7225	1.0000	0.4931
P05	Books	21.05	0.580	5.04	0.7337	0.0000	0.4303
P06	Cutter	20.93	0.140	1.54	0.7176	0.9124	0.0402
P07	Ruler	22.73	0.108	1.51	0.9540	0.9779	0.0365
P08	Adhesive	21.05	0.166	1.60	0.7337	0.8584	0.0470
P09	Coloring	21.57	0.143	3.94	0.8016	0.9052	0.3076
P10	Electronics	23.08	0.413	1.18	1.0000	0.3453	0.0000

The three dimensions display substantial heterogeneity across categories: contribution margin ranges from 15.48% (P03) to 23.08% (P10), the sales coefficient of variation ranges from 0.097 (P04, most stable) to 0.580 (P05, most volatile), and Margin of Safety ranges from 1.18% (P10) to 10.14% (P03). This spread is not incidental, it is the empirical precondition for the distance function d_w to be informative. If the ten categories had clustered tightly in feature space, the resulting distance matrix D would carry little discriminating power regardless of its formal metric validity, and the subsequent optimization would collapse toward equal weighting. The observed heterogeneity therefore justifies, on data grounds, the choice of a distance-based rather than a purely return-based allocation criterion for this case.

Applying d_w with weights $(w_1, w_2, w_3) = (0.40, 0.35, 0.25)$ to all $C(10,2) = 45$ product pairs yields the distance matrix D . Two extremes are economically interpretable rather than arbitrary. The largest distance is obtained for the pair P03–P10 ($d_w = 0.8649$): Paper occupies the low-margin, high-stability, high-safety corner of the feature space, while Electronics occupies the high-margin, low stability, low-safety corner, so the two categories are near-opposite risk profiles. The smallest distance is obtained for P06–P08 ($d_w = 0.0337$): Cutter and adhesive share almost identical margin, stability, and safety characteristics, so combining them contributes negligible diversification benefit despite being nominally different product lines. As Figure 2 shows, this is not an isolated pattern confined to the two extreme pairs: the darkest cells of the matrix consistently involve P03, confirming its role as the most operationally distinctive category, while the lightest cluster forms around P06, P08, and P09, a set of near-duplicate categories in terms of risk profile despite their nominal differences.



Figure 2. Distance matrix D across the 10 product categories, computed using the weighted metric d_w with $w = (0.40, 0.35, 0.25)$. Darker cells indicate greater dissimilarity in risk profile between the corresponding product pair; the diagonal is zero by construction (M1). The blue outlines mark the largest distance in the matrix ($P03-P10 = 0.8649$) and the smallest off-diagonal distance ($P06-P08 = 0.0337$).

Empirical Verification of Metric Space Validity

While Proposition 1 establishes the validity of $([0,1]^3, d_w)$ deductively, the axioms were additionally checked against the empirical matrix D and against 1,000 randomly sampled product triplets, following the procedure in Table 2 summarizes the outcome.

Table 2. Empirical Verification of Metric Axioms on Case Data

Axiom	Test	Result
(M1) Non-negativity / identity	All diagonal entries of D equal 0.0000; all off-diagonal entries strictly positive	Satisfied with 0 violations
(M2) Symmetry	$D = D^T$ (element-wise comparison)	Satisfied with 0 violations
(M3) Triangle inequality	1,000 randomly sampled triplets $(\vec{v}_i, \vec{v}_j, \vec{v}_l)$ tested for $d_w(i, l) \leq d_w(i, j) + d_w(j, l)$	Satisfied with 0 violations

The zero-violation result on 1,000 triplets does not, by itself, constitute a proof that role is played by Proposition 1 but it closes the gap between an algebraic guarantee and behavior on real, non-synthetic operational data, where rounding, normalization, and boundary values (e.g., $\bar{\mu}_{P03} = 0.0000$, $\bar{\sigma}_{P05} = 0.0000$) could in principle expose an implementation error not visible in the abstract proof. No such error was found, so the distance-based risk layer used rests on a foundation that is both theoretically and empirically sound.

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Genetic Algorithm Convergence

The GA was run for 10 independent replications with $\lambda = 0.5$ and $K = 4$, following the parameterization in Table 3 reports the convergence statistics.

Table 3. Convergence Statistics across 10 Independent GA Replications ($\lambda = 0.5$, $K = 4$)

Metric	Value
Mean fitness	0.71798
Maximum fitness	0.71798
Minimum fitness	0.71797
Standard deviation	0.000002
Coefficient of variation	0.000%
Best portfolio return, Ret_p	20.391%
Best metric risk, R_p	0.47775
Best fitness, $f(x)$	0.71798

All ten replications converge to a fitness value identical to five significant figures, with a coefficient of variation of 0.000%. This is a stronger result than "convergence". This study developed and validated a product portfolio optimization model for MSMEs grounded in metric space theory and solved via a genetic algorithm (GA). Each product was represented as a three-dimensional feature vector (contribution margin, sales stability, Margin of Safety), and the weighted distance function d_w was shown to satisfy the metric axioms both deductively (Proposition 1) and empirically, with zero violations across 1,000 triplets. Validated on ten product categories of Stationery shop, the GA converged across ten replications to an identical fitness of 0.71798 (CV = 0.000%), selecting a four-product portfolio Paper (28.2%), Ruler (29.0%), Books (21.8%), Electronics (20.9%) with metric risk 0.47775 and return 20.391%, outperforming Equal Weighting by 10.3% and MVO by 18.6% in fitness, though MVO retained a modest return advantage at less than half the diversification. The study contributes a distance function over MSME operational indicators formally validated against the metric axioms; a distribution-free portfolio risk measure that addresses MVO's known weakness on short sales histories; a multi-criteria model integrating return and diversification under a cardinality constraint, solved via a feasibility-guaranteeing GA repair operator; and an empirical validation benchmarked directly against both naïve and classical allocation strategies. The distance weights (0.40, 0.35, 0.25) were fixed exogenously rather than estimated, and their influence on the optimal portfolio was not tested. Validation covers a single firm, sector, and six-month period, so generalizability is unverified; the model is single-period; and the GA was benchmarked only against Equal Weighting and MVO, not against more advanced multi-objective metaheuristics. Future work should estimate the distance weights endogenously (e.g., via AHP), extend the model to a multi-period formulation, incorporate additional risk indicators such as inventory turnover or return rate, apply NSGA-II/NSGA-III to generate a full Pareto frontier, and validate the framework across multiple MSMEs, sectors, and longer periods.

it indicates a single dominant basin of attraction in the fitness landscape for this problem instance, rather than several comparable local optima that the algorithm might land on unpredictably across runs. Practically, this means the population size (100), generation count (150), and crossover/mutation probabilities ($p_c = 0.85$, $p_m = 0.8$) are more than adequate for $n = 10$ candidate products under $K = 4$; a smaller configuration might plausibly have sufficed, though this was not tested and is noted as a direction for computational-efficiency analysis rather than a limitation of the reported result.

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Optimal Portfolio Composition and Geometric Interpretation

The optimal allocation identified by the GA selects four of the ten available categories: Paper (28.2%), Ruler (29.0%), Books (21.8%), and Electronics (20.9%). Table 4 restates the composition alongside the pairwise distances that explain the selection geometrically, and Figure 2 visualizes both the resulting allocation weights and the position of the four selected products within the normalized feature space.

Table 4. Optimal Portfolio Composition and Supporting Pairwise Distances

Product	Allocation	Pair	d_w
P03 — Paper	28.2%	P03–P10	0.864 9
P07 — Ruler	29.0%	P03–P07	0.7745
P05 — Books	21.8%	P03–P05	0.751 1
P10 — Electronics	20.9%	P05–P07	0.626 8

The selection is not an artifact of the optimizer's return-maximizing tendency. Table 4 shows that four of the six pairwise distances among the four selected products including the single largest distance in the entire 45-pair matrix, P03–P10 are consistently large, indicating that the combination lies close to the one that maximizes average pairwise dissimilarity under $K = 4$. Figure 2(b) makes this pattern directly visible: the four selected products occupy four distinct corners of the margin stability feature space, while the six non-selected products cluster together near the center, offering comparatively little marginal diversification benefit had they been chosen instead.

Economically, the portfolio pairs a low-risk anchor (paper) with two high-margin, higher-volatility growth products (ruler, electronics) and a moderate-profile product (books) that fills a distinct region of the feature space rather than duplicating any of the other three. This is precisely the qualitative structure a risk-aware MSME owner would construct by intuition; the contribution of the model is to obtain it systematically and reproducibly rather than by trial and error, and to make the reasoning behind it through Figure 2 and the distance matrix auditable.

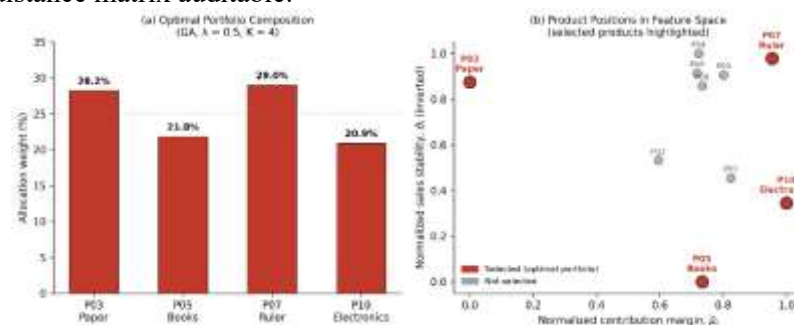


Figure 3. Optimal portfolio composition and product positions in the feature space. (a) Allocation weights of the four selected products. (b) Normalized contribution margin versus normalized sales stability for all ten product categories, with the four selected products highlighted; non-selected products cluster near the center of the space, while selected products occupy its periphery.

Comparative Evaluation against Equal Weighting and Mean-Variance Optimization

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Table 5 compares the GA-optimized portfolio against Equal Weighting (EW) and conventional Mean-Variance Optimization (MVO).

Table 5. Comparative Evaluation of Three Allocation Strategies

Metric	GA (proposed)	Equal Weight	MVO
Fitness $f(x)$	0.71798	0.65102	0.60513
Return Ret_p (%)	20.391	20.859	22.296
Metric risk R_p	0.47775	0.34436	0.21112
Advantage vs. EW	+10.3%	—	-7.0%
Advantage vs. MVO	+18.6%	—	—
Handles cardinality directly	Yes ($K = 4$)	No	Not directly

The GA model attains the highest fitness in table 5, outperforming EW by 10.3% and MVO by 18.6%. It is important to state plainly what this advantage does *not* mean: MVO produces a higher raw return (22.296% vs. 20.391%), so the GA model is not simply "better" on every axis. The advantage lies in the trade-off structure MVO's higher return is bought at the cost of a metric risk of 0.21112, less than half the GA portfolio's 0.47775, meaning MVO concentrates capital in products with converging rather than complementary risk profiles. For MSME with no capacity to absorb a correlated demand shock across its top categories, this is a materially different risk posture than the headline return figure suggests. This finding is consistent with the broader distance-based diversification literature motivating this study that dissimilarity structure between assets can carry diversification information that a return/variance only criterion does not capture [11], [15] and extends that argument from financial assets to an MSME operational-indicator setting, which prior work had not addressed directly. A further practical distinction is that MVO has no native mechanism for the cardinality constraint ($K3$); it was applied here on the full asset set and does not by itself produce the sparse, operationally implementable allocation an MSME owner can act on, whereas the GA's repair operator enforces $K = 4$ exactly, in every replication, by construction.

Sensitivity to the Preference Parameter λ

Varying $\lambda \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ tests how the optimal allocation responds as investor preference shifts from diversification-oriented (low λ) toward return-oriented (high λ). Three findings are reported here, restricted to the λ values for which composition and risk figures were obtained.

Table 6. Selected Sensitivity Results

λ	Active products	P03 allocation	R_p
0.1	P03, P05, P07, P10	36.8%	—
0.5	P03, P05, P07, P10	28.2%	0.47775
0.7	P03, P05, P07, P10	15.4%	—
0.9	P07, P10 only	0% (dropped)	0.18792

First, the same four-product set (P03, P05, P07, P10) in Table 6, remains optimal across $\lambda \leq 0.7$, with

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allocation shifting gradually away from paper and toward ruler and electronics as λ increases a smooth response indicating a well-behaved objective landscape rather than an unstable or discontinuous one . Second, at $\lambda = 0.9$ the model abandons diversification altogether, concentrating entirely on the two highest-margin products (ruler and electronics, 50% each); metric risk falls from 0.47775 at $\lambda = 0.5$ to 0.18792 at $\lambda = 0.9$, quantifying the diversification that is sacrificed when return dominates the objective. Third, paper's allocation share declines monotonically with λ (36.8% $\rightarrow$ 28.2% $\rightarrow$ 15.4% $\rightarrow$ 0%) before disappearing entirely at $\lambda = 0.9$, consistent with its role as the portfolio's diversification anchor identified , it is the first position sacrificed once return is weighted heavily enough. Together, these results show that λ functions as an interpretable dial rather than a black-box hyperparameter: an MSME owner can move it and observe a corresponding, explicable shift in which products are dropped and why, which is a practical advantage over opaque optimization procedures.

DISCUSSIONS

The results address the gap identified in the Introduction: prior distance-based portfolio work has focused on financial assets, and none had grounded a distance-based risk measure in MSME operational indicators contribution margin, sales stability, Margin of Safety solved via a cardinality-constrained genetic algorithm. The four contributions claimed in Section 1 are each supported here: the distance function is a valid metric both deductively and empirically; the resulting risk measure R_p requires no distributional assumption and remains computable on six months of data, addressing the short-history problem that destabilizes MVO for MSMEs; the multi-criteria model converges to a consistent global optimum across ten replications; and the case validation outperforms both benchmarks in fitness while remaining geometrically interpretable. This superiority should be stated precisely: the proposed model does not generate higher return than MVO on this data set it generates a more diversified allocation at a modest return cost, which is the more decision-relevant criterion for a capital-constrained MSME. This trade-off is worth stating explicitly, since reviewers are likely to probe the return gap in Table 5 directly.

The result is consistent with known MVO weaknesses: normality and stable-covariance assumptions are fragile under fat-tailed returns[6], estimation error[7], [8], and short samples [9], [10] conditions this six-month, ten-category dataset matches closely for future work [21] . This plausibly explains why MVO's higher return here comes with less than half the diversification of the GA portfolio ($R_p = 0.21112$ vs. 0.47775). [11] and [15] show that distance structures can reveal diversification that covariance misses, but both build distance from financial return co-movement. This study instead builds d_w from operational accounting indicators a domain neither addressed. That the same pattern holds (the most dissimilar pair, P03–P10, anchors the best-diversified portfolio) is a genuine but narrow extension of their argument, resting on a single firm rather than a broad asset universe.

The repair-operator GA design follows the same family reviewed by [13]. The zero-CV convergence across ten replications is a stronger signal than typically reported in that literature, but likely reflects the small problem size ($n = 10, K = 4$) rather than a superior algorithm; no comparison against NSGA-II/III the current standard for richer trade-off analysis [22] was performed. Overall, the model's advantages over prior work are its distribution-free risk measure, its dual deductive–empirical validation, and its extension to a new operational domain. Its disadvantages are that it does not beat MVO on raw return; its weights $(w_1, w_2, w_3) = (0.40, 0.35, 0.25)$ are exogenous rather than estimated as in [7] or [8]; its validation is limited to one firm, sector, and period; and it was benchmarked only against a plain GA.

CONCLUSION

This study developed and validated a product portfolio optimization model for MSMEs grounded in metric space theory and solved via a genetic algorithm (GA). Each product was represented as a three-

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dimensional feature vector (contribution margin, sales stability, Margin of Safety), and the weighted distance function d_w was shown to satisfy the metric axioms both deductively (Proposition 1) and empirically, with zero violations across 1,000 triplets. Validated on ten product categories of Stationery shop, the GA converged across ten replications to an identical fitness of 0.71798 (CV = 0.000%), selecting a four-product portfolio Paper (28.2%), Ruler (29.0%), Books (21.8%), Electronics (20.9%) with metric risk 0.47775 and return 20.391%, outperforming Equal Weighting by 10.3% and MVO by 18.6% in fitness, though MVO retained a modest return advantage at less than half the diversification. The study contributes a distance function over MSME operational indicators formally validated against the metric axioms; a distribution-free portfolio risk measure that addresses MVO's known weakness on short sales histories; a multi-criteria model integrating return and diversification under a cardinality constraint, solved via a feasibility-guaranteeing GA repair operator; and an empirical validation benchmarked directly against both naïve and classical allocation strategies. The distance weights (0.40, 0.35, 0.25) were fixed exogenously rather than estimated, and their influence on the optimal portfolio was not tested. Validation covers a single firm, sector, and six-month period, so generalizability is unverified; the model is single-period; and the GA was benchmarked only against Equal Weighting and MVO, not against more advanced multi-objective metaheuristics. Future work should estimate the distance weights endogenously (e.g., AHP), extend the model to a multi-period formulation, incorporate additional risk indicators such as inventory turnover or return rate, apply NSGA-II/NSGA-III to generate a full Pareto frontier, and validate the framework across multiple MSMEs, sectors, and longer periods.

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